Lecturered By Dr. H.K. Yadav
Deptt. of Mathematics
Marwari college, Darbhanga.
B.Sc. part. I(H), paper-I, Date - 07-06-2020
Algebra. ~~Group~~. Rings.

Q.1. Define Ring with an example?

Soln: Definition of Ring: → A non-empty set R with two binary operations addition and multiplication denoted by '+' and '·' respectively is called a ring, if it satisfies the following postulates:

(I) $a+b \in R$, $\forall a, b \in R$; [closure law for addition]
II $(a+b)+c = a+(b+c)$, $\forall a,b,c \in R$ [associative law for addition]
III R has an identity, to be denoted by 0, with respect to addition,
i.e. $a+0 = 0+a = a$, $\forall a \in R$
(IV) For every $a \in R$ there exists an element $x \in R$ such that
$a+x = x+a = 0$ [additive inverse]
(V) $a+b = b+a$, $\forall a, b \in R$, [Commutative law for addition]
VI $a \cdot b \in R$, $\forall a, b \in R$ [closure law for multiplication]
(VII) $(a \cdot b) \cdot c = a(b \cdot c)$, $\forall a,b,c \in R$ [Associative law for multiplication]
(VIII) $a \cdot (b+c) = a \cdot b + a \cdot c$
$(b+c) \cdot a = b \cdot a + c \cdot a$ $\forall a, b, c \in R$ [distributive laws]

For example: The set of real numbers is a ring with ordinary addition and multiplication.
The set of all even integers is a Commutative ring.

B.Sc. Part I (H), paper-I, 07-05-2020

Q.2. To prove that for all a, b, c in a ring R

(I) $a0 = 0a = 0$ (II) $a(-b) = -(ab) = (-a)b$

(III) $(-a)(-b) = ab$ (IV) ~~a(b-c) = ab-ac (V) (b-c)a = ba-ca.~~

Proof: (I) $a0 = a(0+0)$ [$\because 0+0=0$]

$= a0 + a0$ [By left distributive law]

$0 + a0 = a0 + a0$ [By the property of identity]

[$\because 0 + a0 = a0 \forall a0 \in R$]

$\Rightarrow 0 = a0$ [By right cancellation law $\because R$ is a group]

Similarly, $0a = (0+0)a$ [$\because 0+0=0$]

$= 0a + 0a$ [By right distributive law]

$\Rightarrow 0a + 0 = 0a + 0a$ [By the property of identity]

$\Rightarrow 0 = 0a$ [By left cancellation law]

Hence, $a0 = 0a = 0$ proved

II $a0 = 0$ [from I]

(II) $\Rightarrow a[(-b)+b] = 0$ [By the property of inverse in R]

$\Rightarrow a(-b) + ab = 0$ [By left distributive law]

$\Rightarrow a(-b) = -(ab)$ [$\because a+b=0 \Rightarrow a = -b$]

Similarly, $0b = 0$ [by I]

$\Rightarrow [(-a)+a]b = 0$ [By the property of inverse in R]

$\Rightarrow (-a)b + ab = 0$ [By right distributive law]

$\Rightarrow (-a)b = -(ab)$ [$\because a+b=0 \Rightarrow a = -b$

Hence, $(-a)b = -(ab) = a(-b)$

Proved

(III) $(-a)(-b) = (ab)$

Since, we have, $a(-b) = -(ab)$

$\therefore (-a)(-b) = -[(-a)]$ [$\because (-a)b = -(ab)$]

$= -[-(ab)]$

$= a$

Hence, $(-a)(-b) = (ab)$ Proved.

Scanned with CamScanner